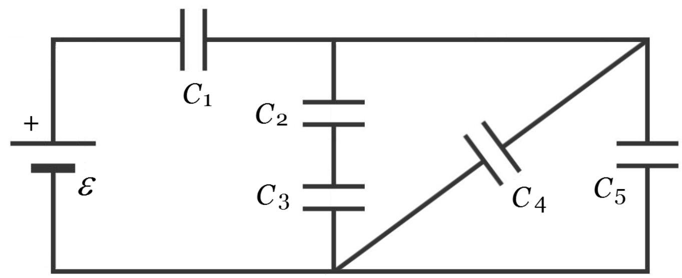
PHYS 2212
Recitation 06 March 1–4

Example
In the circuit shown, $C_1 = C_4 = C_5 = C$ and $C_2 = C_3 = 2C$.

a) Find the equivalent capacitance of the circuit.

b) Find the energy stored in capacitor C_4. Express your answers in terms of C and $\mathcal{E}$.



PHYS 2212 Section: ______ Name: ____________________

Recitation 06 March 1–4

Problem

In the circuit shown, which is subtly different from the circuit in the example, $C_1 = C_4 = C_5 = C$ and $C_2 = C_3 = 2C$.

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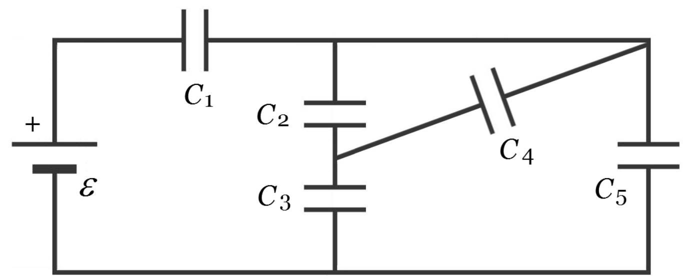

PHYS 2212 **Solution**
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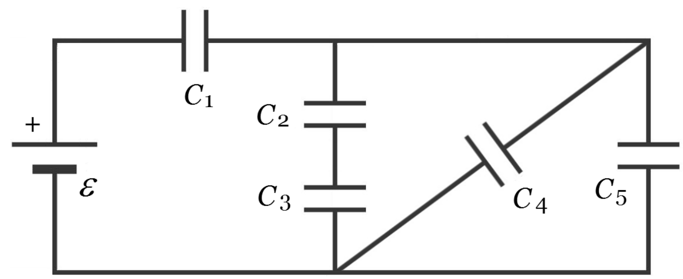

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a) In order to find the equivalent capacitance of this circuit, one must first realize that capacitor C_4 is in parallel with C_5 and the branch containing capacitors C_2 and C_3. This is true because the upper plates of C_2, C_4, and C_5 are connected by ideal wire and are thus at the same potential. The same is true of the bottom plates of C_3, C_4, and C_5. Given this fact, one can begin reducing the circuit, starting with the two series capacitors C_2 and C_3.

$$C_{23} = \left[\frac{1}{C_2} + \frac{1}{C_3}\right]^{-1} = \left[\frac{1}{2C} + \frac{1}{2C}\right]^{-1} = C$$

Next are the parallel capacitors C_{23}, C_4, and C_5.

$$C_{2345} = C_{23} + C_4 + C_5 = C + C + C = 3C$$

And finally the series capacitors C_1 and C_{2345}.

$$C_{\text{eq}} = \left[\frac{1}{C_1} + \frac{1}{C_{2345}}\right]^{-1} = \left[\frac{1}{C} + \frac{1}{3C}\right]^{-1} = \frac{3}{4}C$$

b) In order to find the energy stored in capacitor C_4 it is necessary to find the potential difference ΔV_4 across it. However, because C_{23}, C_4, and C_5 are in parallel, the potential difference across each must be equal to the potential difference V_{2345} across their equivalent capacitor C_{2345}. Furthermore, since C_1 and C_{2345} are in series, they must hold the same charge, which is the same as the charge on the equivalent capacitor. Thus:

$$\Delta V_4 = \Delta V_{2345} = \frac{Q_{\text{eq}}}{C_{2345}} = \frac{3C\mathcal{E}/4}{3C} = \mathcal{E}/4$$

So the energy stored in capacitor C_4 is:

$$U_4 = \frac{1}{2}C_4\left(\Delta V_4\right)^2 = \frac{1}{2}C\left(\mathcal{E}/4\right)^2 = \frac{C\mathcal{E}^2}{32}$$

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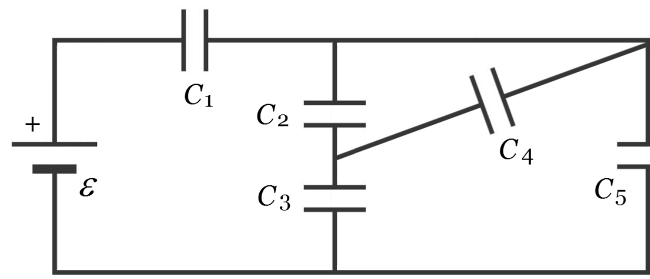
a) Find the equivalent capacitance of the circuit.

b) Find the energy stored in capacitor C_4. Express your answers in terms of C and $\mathcal{E}$.



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a) The only difference between this problem and the previous one is that capacitor $C4$ is now in parallel with C_2 only. The equivalent capacitor C_{24} is then in series with C_3.

$$C_{24} = C_2 + C_4 = 2C + C = 3C \qquad \text{and} \qquad C_{234} = \left[\frac{1}{C_{24}} + \frac{1}{C_3}\right]^{-1} = \left[\frac{1}{3C} + \frac{1}{2C}\right]^{-1} = \frac{6}{5}C$$

Those two are in parallel with C_5.

$$C_{2345} = C_{234} + C_5 = \frac{6}{5}C + C = \frac{11}{5}C$$

Finally, capacitors C_1 and C_{2345} are in series, making the equivalent capacitance of the circuit:

$$C_{\text{eq}} = \left[\frac{1}{C_1} + \frac{1}{C_{2345}}\right]^{-1} = \left[\frac{1}{C} + \frac{5}{11C}\right]^{-1} = \frac{11}{16}C$$

b) In order to find the energy stored in capacitor C_4 it is necessary to find the potential difference ΔV_4 across it. Starting with the equivalent capacitance, we can find the charge delivered by the battery.

$$Q_{\text{eq}} = C_{\text{eq}}\mathcal{E} = \frac{11}{16}C\mathcal{E}$$

Since C_1 and C_{2345} are in series, this is also the charge on each of them. The potential difference ΔV_{2345} can be found. Since C_{234} and C_5 are in parallel, this will be the potential difference across each of them.

$$\Delta V_{234} = \Delta V_{2345} = \frac{Q_{\text{eq}}}{C2345} = \frac{11C\mathcal{E}/16}{11C/5} = \frac{5\mathcal{E}}{16}$$

The charge on C_{234}, then, is

$$Q_{234} = C_{234}\,\Delta V_{234} = \left(\frac{6C}{5}\right)\left(\frac{5\mathcal{E}}{16}\right) = \frac{3C\mathcal{E}}{8}$$

Since C_{24} and C_3 are in series, this is the charge on each of them. ΔV_{24} can then be found.

$$Q_{24} = Q_{234} \qquad \text{so} \qquad \Delta V_{24} = \frac{Q_{234}}{C_{24}} = \frac{3C\mathcal{E}/8}{3C} = \frac{\mathcal{E}}{8}$$

Since C_2 and C_4 are in parallel, this is also the potential difference across C_2, so

$$U_4 = \frac{1}{2}C_4\left(\Delta V_4\right)^2 = \frac{1}{2}C\left(\frac{\mathcal{E}}{8}\right)^2 = \frac{C\mathcal{E}^2}{128}$$